Fault Tolerant Strategy for Semi-Active Suspensions with LPV Accommodation

Juan C. Tudon-Martinez, Sébastien Varrier, Olivier Sename, Ruben Morales Menendez, John Jairo Martinez Molina, Luc Dugard

Fault Tolerant Strategy for Semi-Active Suspensions with *LPV* Accommodation *

Juan C. Tudón-Martínez[1], Sébastien Varrier[1], Olivier Sename[1]
Ruben Morales-Menendez[2], John-Jairo Martinez[1] and Luc Dugard[1]

***Abstract*— A novel fault tolerant strategy to compensate multiplicative actuator faults (damper oil leakages) in a semi-active suspension system is proposed. The compensation of the lack of damping force caused by a faulty damper is carried on by the remainder three healthy semi-active dampers. Once a faulty damper is detected and isolated by a *Fault Detection and Isolation* strategy based on parity-space, an estimator is activated to compute the missing damping force to compensate. In order to fulfill the semi-active damper constraints, the fault accommodation is based on the Linear-Parameter Varying (*LPV*) control strategy. Thus, each corner has a fault estimator and an *LPV* controller oriented to comfort and road holding. Simulation results show that the proposed fault tolerant semi-active suspension improves the vehicle comfort up to 60% with respect to a controlled suspension without fault-tolerant strategy and 82% with respect to a passive suspension.**

I. Introduction

Because the automotive industry demands safer and more comfortable vehicles constantly, the development of intelligent suspension systems has gained importance during last years. Semi-active dampers are efficient actuators to improve passengers comfort and car road holding as well [1]. However, as it shown in recent studies [1], [2], the control design for such dampers is complex due to the dissipativity constraint on the damper (which is indeed a state dependent saturation [3]).

Semi-active dampers allow the development of novel techniques to model, monitor and control an intelligent suspension system with fault-tolerance features to improve the process reliability. A Fault-Tolerant Controller (*FTC*) in a suspension system is designed to maintain the desired comfort and road holding performances as much as possible when a fault occurs. There are two main groups of *FTC*: passive (off-line designed) and active (on-line control reconfiguration mechanism); this study is focused on an active *FTC* because of its lower conservatism than a passive approach.

Active systems on-line utilize some fault information to compensate the fault effect by using a Fault Detection and Isolation (*FDI*) strategy that can be model-based or data-based. *FDI* modules based on analytical redundancy are the most accepted strategies for fault estimation [4], in particular parity-space approaches, e.g. [5] presents a fault signature of an intelligent damper by using parity relations.

During last years, the Linear Parameter-Varying (*LPV*) systems have gained importance as a solution to develop embedded control strategies for complex systems, e.g. the non-linear vehicle dynamics, [6]. Recently, *LPV*-based techniques have been incorporated to *FTC* approaches, where additive faults are considered as a varying parameter to be accommodated by a scheduling reconfiguration approach, [7]. Based also on a scheduling reconfiguration, [8] proposed an active *FTC* using Sliding Mode observers. Both proposals lead to interesting results for a full vehicle but they are developed for active suspensions without actuator constraints. For semi-active suspensions, the authors have proposed in [9] an active *FTC* (*LPV*-based) for additive actuator faults in a Quarter of Vehicle (*QoV*) model. The reconfiguration depends on the fault estimation, but a saturation constraint is needed.

As an extension of [9], this paper proposes an active *FTC* for a full vehicle semi-active suspension, when one of the dampers suffers an oil leakage (multiplicative actuator fault). The main contribution is the development of a new 2-level reconfiguration strategy that compensates a damper leakage by using the healthy dampers; because the force is inherently limited by the shock absorbers, an *LPV* control design includes a saturation constraint on the compensation force used to the fault accommodation. At a higher level, 1) an *FDI* module identifies the faulty damper by using a parity-space approach and 2) the compensation forces are computed. At a lower level, each corner has a fault estimator and an *LPV* based controller which also satisfies the dissipativity constraint of a commercial Magneto-Rheological (*MR*) damper.

The paper is organized as follows: the problem statement is described in the next section. Sections III and IV describe the fault detection and estimation methodology and the *FTC* design, respectively. Section V presents the simulation results. Finally, conclusions are presented in section VI.

II. Problem Statement

A seven degrees of freedom model of the vertical dynamics of a full vehicle, including a semi-active suspension system, is considered as [10]:

$$
\begin{aligned}
m_s\ddot{z}_{s_c} &= -\left(F_{s_1}+F_{s_2}+F_{s_3}+F_{s_4}\right) &\text{(1a)}\\
I_{xx}\ddot{\theta} &= \left(F_{s_1}-F_{s_2}\right)t_f+\left(F_{s_3}-F_{s_4}\right)t_r &\text{(1b)}\\
I_{yy}\ddot{\phi} &= \left(F_{s_4}+F_{s_3}\right)l_r-\left(F_{s_2}+F_{s_1}\right)l_f &\text{(1c)}\\
m_{us_i}\ddot{z}_{us_i} &= F_{s_i}-F_{t_i} &\text{(1d)}
\end{aligned}
$$

* This work was partially supported by CONACyT through the PCP project 03/10 FLORE and the French national project INOVE/ANR 2010 BLAN 0308.

[1]J.C. Tudón-Martínez, S. Varrier, O. Sename, J.J. Martínez and L. Dugard are with Gipsa-Lab, 11 rue des mathématiques, 38402 Grenoble, France juan-carlos.tudon@gipsa-lab.grenoble-inp.fr

[2]R. Morales-Menendez is with Tecnológico de Monterrey, Campus Monterrey, Av. Garza Sada 2501, 64849 Monterrey NL, México rmm@itesm.mx

where the vertical suspension force at each corner (F_{s_i}, with $i = 1, \ldots, 4$) is composed by the spring (stiffness k_{s_i}) and the semi-active damping force (F_{sa_i}), as:

$$F_{s_i} = k_{s_i}\left(z_{s_i} - z_{us_i}\right) + F_{sa_i} \tag{2}$$

while the tire vertical force, with stiffness k_{t_i}, is given by:

$$F_{t_i} = k_{t_i}\left(z_{us_i} - z_{r_i}\right) \tag{3}$$

The semi-active force is modeled following [11], [12], as:

$$F_{sa_i} = \underbrace{b_1\left(\dot{z}_{s_i} - \dot{z}_{us_i}\right) + b_2\left(z_{s_i} - z_{us_i}\right)}_{F_{p_i}} + F_{I_i}(I) \tag{4a}$$

$$F_{I_i}(I) = I \cdot f_c \cdot \tanh\left[a_1\left(\dot{z}_{s_i} - \dot{z}_{us_i}\right) + a_2\left(z_{s_i} - z_{us_i}\right)\right] \tag{4b}$$

where I is the electric current and F_{I_i} is the controlled semi-active force to improve the comfort and/or maintain the road holding. Note that for $I = 0$, F_{sa_i} reduces to the passive damping force F_{p_i} of the suspension system. The sprung mass positions and velocities at each corner can be linearly derived from the vehicle dynamics, at small ϕ and θ, as:

$$\begin{array}{llll} z_{s_1} \approx & z_{s_c} + l_f\phi - t_f\theta & z_{s_3} \approx & z_{s_c} - l_r\phi - t_r\theta \\ z_{s_2} \approx & z_{s_c} + l_f\phi + t_f\theta & z_{s_4} \approx & z_{s_c} - l_r\phi + t_r\theta \end{array} \tag{5}$$

$$\begin{array}{llll} \dot{z}_{s_1} \approx & \dot{z}_{s_c} + \dot{\phi}l_f - \dot{\theta}t_f & \dot{z}_{s_3} \approx & \dot{z}_{s_c} - \dot{\phi}l_r - \dot{\theta}t_r \\ \dot{z}_{s_2} \approx & \dot{z}_{s_c} + \dot{\phi}l_f + \dot{\theta}t_f & \dot{z}_{s_4} \approx & \dot{z}_{s_c} - \dot{\phi}l_r + \dot{\theta}t_r \end{array} \tag{6}$$

The variables and parameters of this model are described in Table I. The corresponding parameters were obtained from a commercial light-duty vehicle and *MR* damper.

TABLE I

VERTICAL FULL VEHICLE PARAMETERS AND VARIABLES.

Parameter/Variable	Description	Value
$a_{1,2}$	*MR* damping coefficients	37.8 s/m & 22.1 m^{-1}
$b_{1,2}$	Passive damping coefficients	2,830 Ns/m & -7,897 N/m
f_c	*MR* dynamic yield force	600.9 N/A
I_{xx}, I_{yy}	Roll and pitch inertia	844 & 4434 Kg · m^2
$k_{s_{1,2}}$, $k_{s_{3,4}}$	Front and rear spring stiffness	43,500 & 39,300 N/m
$k_{t_{1,2,3,4}}$	Tire stiffness	230,000 N/m
l_f	Distance *COG** - front track	1.56 m
l_r	Distance *COG** - rear track	2.11 m
m_s	Total sprung mass	2,032 Kg
$m_{us_{1,2}}$	Front unsprung masses	81.5 Kg
$m_{us_{3,4}}$	Rear unsprung masses	140 Kg
t_f	Distance *COG**-front left tire	0.85 m
t_r	Distance *COG**-rear left tire	0.85 m
F_{p_i}	Passive damping forces	$i = 1, \ldots, 4$
F_{sa_i}	Semi-active damping forces	$i = 1, \ldots, 4$
F_{s_i}	Suspension forces	$i = 1, \ldots, 4$
F_{t_i}	Tire forces	$i = 1, \ldots, 4$
z_{r_i}	Ground vertical positions	
z_{us_i}, $\dot{z}_{us_i}$, $\ddot{z}_{us_i}$	Position, velocity, acc of m_{us_i}	
z_{s_i}, $\dot{z}_{s_i}$	Position, velocity of m_{s_i}	
$\ddot{z}_{s_c}$	*COG** acceleration of m_s	
$\ddot{\theta}$, $\ddot{\phi}$	Roll & Pitch acc of m_s	

* *COG* Center Of Gravity

Problem definition. Consider a fault on a semi-active damper, e.g. an oil leakage, which induces a lack of force modeled as:

$$\overline{F}_{sa_i} = \alpha F_{sa_i} = F_{sa_i}(1 - \delta) = F_{sa_i} - F_{\delta_i} \tag{7}$$

where $\overline{F}_{sa_i}$ is the faulty force given as a reduction of the nominal force and, $\alpha \in [0, 1)$ is the oil leakage degree, e.g. $\alpha = 0.7$ means 70% of F_{sa_i} due to a lost force F_{δ_i} of 30%.

The objective of the *FTC* strategy is to avoid a propagation of the fault effects into the whole vertical dynamics (deterioration of comfort and road holding) by designing a reconfiguration system based on the remaining healthy dampers. The main idea is to compute the required force at each corner that compensates the faulty damper effect, such that:

$$\begin{pmatrix} \ddot{z}_s \\ \ddot{\theta} \\ \ddot{\phi} \end{pmatrix} = f\left(\overline{F}_{sa_i}, F_{n_j}, F_{c_j}\right) \text{ for } \begin{cases} i \in \{1,2,3,4\} \\ j \to 3 \text{ integers} \\ \quad \text{with } j \neq i \end{cases} \tag{8}$$

where, F_{c_j} is the corresponding compensation force in the healthy damper j obtained by the equilibrium analysis of the dynamics (1) to accommodate the lack of force in the faulty damper i and, F_{n_j} is the *nominal damping force* (suspension system free of faults). Thus, the semi-active damping force in the healthy dampers will be:

$$F_{sa_j} = F_{p_j} + \underbrace{F_{n_j} + F_{c_j}}_{F_{I_j}(I_n, I_c)} \tag{9}$$

where, $F_{I_j}(I_n, I_c)$ is obtained by an *LPV* controller, whose design fulfills with the semi-activeness and saturation of the damper when the compensation force F_{c_j} is added, such that the closed-loop system be:

$$\begin{aligned} \dot{x} = & \ \mathcal{A}(\rho) \cdot x + \mathcal{B}_1 \cdot \underbrace{(I_n + I_c)}_{\mathcal{K}_{LPV}(\rho)\cdot x} + \mathcal{B}_2 \cdot z_r \\ y = & \ \mathcal{C}(\rho) \cdot x + \mathcal{D}_1 \cdot (I_n + I_c) + \mathcal{D}_2 \cdot z_r \end{aligned} \tag{10}$$

with $\mathcal{K}_{LPV}(\rho) = \sum_{i=1}^{N} \xi_i(\rho)\mathcal{K}_i$ by appropriately choosing the gains $\mathcal{K}_i$, $i = 1, \ldots, N$, such that the closed-loop system ($\mathcal{A}, \mathcal{B}_1, \mathcal{B}_2, \mathcal{C}, \mathcal{D}_1, \mathcal{D}_2$) is asymptotically stable for all parameter variations; two varying parameters are used in the controller design: ρ_1^* represents the nonlinearities of the *MR* damper and ρ_2^* is used for the actuator saturation that depends on the maximum damping force available for the compensation.

III. FAULT DETECTION AND ISOLATION

The isolation of the faulty damper and the estimation of the damper fault F_{δ_i} is based on the parity-space approach because of its applicability to nonlinear systems, speed of detection, isolability property, robustness and computational complexity [4]. The aim is to synthesize a residual sensitive to the fault F_{δ_i} and insensitive to bounded exogenous inputs, e.g. the road profile or load transfers on the vehicle.

A. Modeling of the system

By coupling the dynamics of the full vehicle model (1), (2), (3), (5) and (6) with the faulty damper (4) and (7), a state-space representation system with 14 states is handled as:

$$\dot{x}_f = A_f x_f + B_{fI} I + B_{fr} z_r + B_{fF} F_\delta \tag{11a}$$

$$y_f = C_f x_f + D_{fI} I + D_{fr} z_r + D_{fF} F_\delta \tag{11b}$$

where, $x_f = [z_{s_c}, z_{us_1}, z_{us_2}, z_{us_3}, z_{us_4}, \theta, \phi, \dot{z}_{s_c}, \dot{z}_{us_1}, \dot{z}_{us_2}, \dot{z}_{us_3}, \dot{z}_{us_4}, \dot{\theta}, \dot{\phi}]^T$ are the full system states, the input is $I = [I_1, I_2, I_3, I_4]^T$, $z_r = [z_{r_1} z_{r_2} z_{r_3}, z_{r_4}]^T$ is the road profile and $F_\delta = [F_{\delta_1}, F_{\delta_2}, F_{\delta_3}, F_{\delta_4}]^T$ is the fault vector. The measurement vector y_f is given by 8 accelerometers as $y_f = [\ddot{z}_{s_1}, \ddot{z}_{s_2}, \ddot{z}_{s_3}, \ddot{z}_{s_4}, \ddot{z}_{us_1}, \ddot{z}_{us_2}, \ddot{z}_{us_3}, \ddot{z}_{us_4},]^T$. A discrete version of the system (11) with a sampling period T_s is:

$$\begin{aligned} x_{f,k+1} &= A_{fd} x_{f,k} + B_{fdI} I_k + B_{fdr} z_{r,k} + B_{fdF} F_{\delta,k} \\ y_{f,k} &= C_f x_{f,k} + D_{fI} I_k + D_{fr} z_{r,k} + D_{fF} F_{\delta,k} \end{aligned} \tag{12}$$

B. Fault detection

The system is nonlinear due to the parameters $\rho_{1_i} = \tanh\left[a_1\left(\dot{z}_{s_i} - \dot{z}_{us_i}\right) + a_2\left(z_{s_i} - z_{us_i}\right)\right]$ in the matrix B_{fI} and B_{fdI}. Since these varying parameters appear in a linear multiplicative way, they are extracted from the system and added to the control input as:

$$\begin{aligned} x_{f,k+1} &= A_{fd}x_{f,k} + \overline{B}_{fdI}\underbrace{\rho_1 I_k}_{\overline{I}_k} + B_{fdr}z_{r,k} + B_{fdF}F_{\delta,k} \\ y_{f,k} &= C_f x_{f,k} + \overline{D}_{fI}\rho_1 I_k + D_{fr}z_{r,k} + D_{fF}F_{\delta,k} \end{aligned} \quad (13)$$

The system (13) is linear and the classical parity space approach is handled to isolate and estimate the fault.

C. Fault isolation

The aim of the fault isolation part is to highlight which damper is faulty. For this purpose, 4 schemes are implemented, one for each corner of the vehicle. Each fault detector has to be sensitive to the fault related to its corner.

The parity-space approach aims at extending the outputs of the system along a horizon s. It leads to:

$$\begin{aligned} &Y_s - G_I U_s = H x_{f,k} + G_r U_r \\ &\quad + G_{F1}F_1 + G_{F2}F_2 + G_{F3}F_3 + G_{F4}F_4 \end{aligned} \quad (14)$$

where $Y_s = \begin{bmatrix} y_{f,k-s} \\ y_{f,k-(s-1)} \\ \vdots \\ y_{f,k} \end{bmatrix}$, $U_s = \begin{bmatrix} \overline{I}_{k-s} \\ \overline{I}_{k-(s-1)} \\ \vdots \\ \overline{I}_k \end{bmatrix}$,

$U_r = \begin{bmatrix} z_{r,k-s} \\ z_{r,k-(s-1)} \\ \vdots \\ z_{r,k} \end{bmatrix}$, $F_i = \begin{bmatrix} F_{\delta_i,k-s} \\ F_{\delta_i,k-(s-1)} \\ \vdots \\ F_{\delta_i,k} \end{bmatrix}$, $H = \begin{bmatrix} C_f \\ C_f A_{fd} \\ \vdots \\ C_f A_{fd}^s \end{bmatrix}$

and $G_i = \begin{bmatrix} D_i & 0 & 0 & \cdots & 0 \\ C_f B_i & D_i & \ddots & \ddots & 0 \\ C_f A_{fd} B_i & C_f B_i & D_i & \ddots & 0 \\ C_f A_{fd}^2 B_i & C_f A_{fd} B_i & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ C_f A_{fd}^{s-1} B_i & C_f A_{fd}^{s-2} B_i & \cdots & C_f B_i & D_i \end{bmatrix}$

where, G_i can be G_I, G_r or G_{F_i}, according to the matrices B_i and D_i associated to the exogenous input in (13).

The presentation of the approach will be focused on the front-left corner (actuator 1) for clarity. The classical approach would consist in finding a parity matrix W_1 making sensitive the residual $r_{1,k} \triangleq W_1(Y_s - G_I U_s)$ only to the fault F_1, i.e. the matrix W_1 has to be orthogonal to the matrix $\begin{bmatrix} H & G_r & G_{F2} & G_{F3} & G_{F4} \end{bmatrix}$. Because this constraint is hard to be satisfied since this matrix contains too many columns, a non-optimal (with non perfect decoupling of the fault) solution has been considered, through the optimization problem:

$$\text{find } W_1 \text{ subject to: } \begin{cases} W_1 G_r = 0 \\ \max\limits_{W_1} W_1 G_{F1} \\ \min\limits_{W_1} W_1 G_{F2} \\ \min\limits_{W_1} W_1 G_{F3} \\ \min\limits_{W_1} W_1 G_{F4} \end{cases} \quad (15)$$

This optimization problem guaranties that r_1 is receptive to the fault, maximizing the effect of the fault F_{δ_1} and minimizing the effect of the others, [9], [13]. Similarly, the approach is implemented into the other three corners.

D. Fault estimator

The last step of the *FDI* procedure is the estimation of the damper fault F_δ. At each corner, a fault estimator is designed. The main advantage of this structure is that the fault estimator is locally activated and not affected by other signals. The parity-space approach leads to:

$$Y_s - G_I U_s = H x_{f,k} + G_r U_r + G_{Fi} F_i \quad (16)$$

where i specifies the faulty damper. Note that in this case, the horizon s could be mandatorily different in the isolation part. The classical parity matrix $W_{i,est}$ for estimation purpose is synthesized so that it is orthogonal to the matrix H and G_r to be insensitive to the road profile.

The residual $r_{i,est,k} \triangleq W_{i,est}\left(Y_s - G_I U_s\right)$ will be sensitive to any fault F_{δ_i}, such that the parity relation $r_{i,est,k} = W_{i,est} G_{Fi} F_i$ be robust to the fault parameter α in eq. (7).

The fault estimator $\hat{F}_{\delta_i}$ is performed by inverting the relation between the residual and the fault. Indeed, it can be written, thanks to the z-transform formalism:

$$\begin{aligned} r_{i,est,k} &= W_{i,est} G_{Fi} F_i \\ &= W_{i,est} G_{Fi} [z^{-s} \quad z^{-(s-1)} \quad \ldots \quad z^{-1} \quad 1]^T F_{\delta_i,k} \\ r_{i,est,k} &= W G_{Fi}(z) F_{\delta_i,k} \end{aligned} \quad (17)$$

Inverting the system (17) leads to:

$$\hat{F}_{\delta_i,k} = \left[W G_{Fi}(z)\right]^{-1} \; r_{i,est,k} \quad (18)$$

IV. Fault Tolerant Semi-Active Suspension

When an oil leakage occurs in one of the dampers and the car is moving, the lack of damping force increases the vertical vehicle body motion. To reduce the fault propagation into the vertical dynamics, eq. (1), an extra damping force is implemented at each corner to compensate for the faulty damper and thus maintain the comfort in the vehicle.

Figure 1 shows a block diagram of the proposed *FTC*. Each corner has an *LPV* controller for comfort and road holding; also, a fault estimator. Once a damper fault is detected and estimated, the reconfiguration module computes the compensation force that the healthy dampers must deliver; the *LPV* based controller design satisfies the *MR* damper constraints (semi-activeness and saturation).

A. Fault Tolerant Control Concept

Consider that only **one** actuator is faulty, e.g. the actuator 1 leading to the damping force:

$$\overline{F}_{sa_1} = F_{sa_1} - F_{\delta_1} = F_{p_1} + F_{n_1} - F_{\delta_1} \quad (19)$$

To compensate this lack of actuation, an additional force to the nominal one is demanded on the remainder of the healthy dampers, such that:

$$F_{sa_{j,j\neq 1}} = F_{p_{j,j\neq 1}} + F_{n_{j,j\neq 1}} + F_{c_{j,j\neq 1}} \quad (20)$$

where the capacity of compensation is limited by the maximum actuation value with a compromise between performance and compensation managed by the controller.

The vehicle dynamics in the *COG*, given in (1), become:

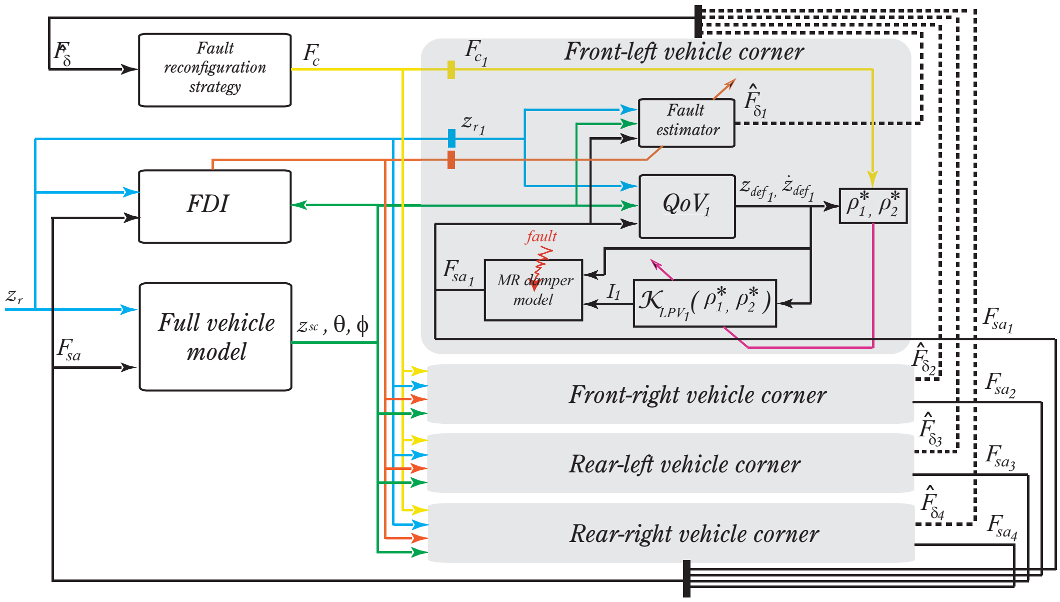

Fig. 1. Block diagram of the proposed Fault Tolerant semi-active suspension system.

$$
\begin{aligned}
m_s\ddot{z}_s &= -(F_{p_1}+F_{n_1}-F_{\delta_1}+F_{p_2}+F_{n_2}+F_{c_2} && \text{(21a)}\\
&\quad +F_{p_3}+F_{n_3}+F_{c_3}+F_{p_4}+F_{n_4}+F_{c_4})\\
I_{xx}\ddot{\theta} &= (F_{p_1}+F_{n_1}-F_{\delta_1}-F_{p_2}-F_{n_2}-F_{c_2})\,t_f && \text{(21b)}\\
&\quad +(F_{p_3}+F_{n_3}+F_{c_3}-F_{p_4}-F_{n_4}-F_{c_4})\,t_r\\
I_{yy}\ddot{\phi} &= (F_{p_4}+F_{n_4}+F_{c_4}+F_{p_3}+F_{n_3}+F_{c_3})\,l_r && \text{(21c)}\\
&\quad -(F_{p_2}+F_{n_2}+F_{c_2}+F_{p_1}+F_{n_1}-F_{\delta_1})\,l_f
\end{aligned}
$$

Based on eq. (21), the compensation of F_{δ_1} by the healthy dampers as $F_{c_j,j\neq 1}$ leads to:

$$F_{\delta_1}-F_{c_2}-F_{c_3}-F_{c_4}=0 \quad \text{(22a)}$$

$$-t_fF_{\delta_1}-t_fF_{c_2}+t_rF_{c_3}-t_rF_{c_4}=0 \quad \text{(22b)}$$

$$l_rF_{c_4}+l_rF_{c_3}-l_fF_{c_2}+l_fF_{\delta_1}=0 \quad \text{(22c)}$$

which can be rewritten in matrix form as:

$$\underbrace{\begin{bmatrix}1\\-t_f\\l_f\end{bmatrix}}_{N}F_{\delta_1}+\underbrace{\begin{bmatrix}-1&-1&-1\\-t_f&t_r&-t_r\\-l_f&l_r&l_r\end{bmatrix}}_{M}\begin{bmatrix}F_{c_2}\\F_{c_3}\\F_{c_4}\end{bmatrix}=0 \quad \text{(23)}$$

M is invertible since $\det(M)=-2t_r(l_f+l_r)\neq 0$. Since F_{δ_1} is not measured but estimated, this allows the straightforward computation of the compensating forces $F_{c_{j,j\neq 1}}$, as:

$$\begin{bmatrix}F_{c_2}\\F_{c_3}\\F_{c_4}\end{bmatrix}=-M^{-1}N\hat{F}_{\delta_1}=\begin{bmatrix}1\\t_f/t_r\\-t_f/t_r\end{bmatrix}\hat{F}_{\delta_1} \quad \text{(24)}$$

To expand the fault tolerance concept for other possible faulty dampers, the full matrix of compensation is given by:

$$\begin{bmatrix}F_{c_1}\\F_{c_2}\\F_{c_3}\\F_{c_4}\end{bmatrix}=\begin{bmatrix}0&1&\frac{t_r}{t_f}&-\frac{t_r}{t_f}\\1&0&-\frac{t_r}{t_f}&\frac{t_r}{t_f}\\\frac{t_f}{t_r}&-\frac{t_f}{t_r}&0&1\\-\frac{t_f}{t_r}&\frac{t_f}{t_r}&1&0\end{bmatrix}\begin{bmatrix}\hat{F}_{\delta_1}\\\hat{F}_{\delta_2}\\\hat{F}_{\delta_3}\\\hat{F}_{\delta_4}\end{bmatrix} \quad \text{(25)}$$

B. LPV/H_∞ Control Design

The control of the vertical dynamics is ensured by the suspension system to achieve frequency specification performances, [2]. Based on [6] that embeds into the control design the constraints of dissipativity and saturation of a semi-active damper with two scheduling parameters, here an *LPV*/H_∞ controller is designed at each corner for: a) performances of comfort and road holding against road disturbances and unmodeled dynamics and, b) compensation of lack of damping force of a faulty damper.

1) Scheduling parameters: Two varying parameters are used in the *LPV* controller synthesis for each corner: ρ_1^* includes the nonlinearities of the damper and is given by,

$$\rho_1^*=\tanh\left(a_1\dot{z}_{def}+a_2z_{def}\right)\cdot\frac{\tanh\left(\frac{I_f}{I_{n0}+I_{c0}}\right)}{\frac{I_f}{I_{n0}+I_{c0}}}\quad\in[-1,1] \quad \text{(26)}$$

where I_{n0} and I_{c0} are the average of electric currents dedicated for the nominal control and compensation. $I_f=I_n+I_c$ is the electric current bounded by the saturation constraint:

$$0\leq I_{min}<I_f\leq I_{max} \quad \text{(27)}$$

ρ_2^* is used directly to saturate the controller output, given by:

$$\rho_2^*=\frac{\tanh(a_1\dot{z}_{def}+a_2z_{def})}{a_1\dot{z}_{def}+a_2z_{def}}\cdot(I_{no}+I_{co})\quad\in[0,\tfrac{I_{max}}{2}] \quad \text{(28)}$$

Note that there is a compromise between the control performances and the compensation because $I_{n0}+I_{c0}=I_{max}/2$; the capability to compensate for a faulty damper is limited by:

$$I_c=\begin{cases}I_c=2\cdot I_{c0}+I_{n0} & if\quad I(F_c)\geq I_{c_{max}}\\ I_{n0}<I_c<I_{c_{max}} & if\quad 0<I(F_c)<I_{c_{max}}\\ I_c=I_{n0} & if\quad I(F_c)<0\end{cases} \quad \text{(29)}$$

where, $I(F_c)=\left[F_c\cdot\coth\left(a_1\dot{z}_{def}+a_2z_{def}\right)\right]/f_c$ is the inverse dynamics of the *MR* damper model, eq. (4b), that depends on the compensation force, eq. (25), and $I_{c_{max}}=2\cdot I_{c0}+I_{n0}$ is the maximum electric current available for compensation. If the faulty damper requires a greater force than the one that could be generated by $I_{c_{max}}$, the fault will not be well accommodated. In the fault free case, $I_{c0}=0$, and the nominal controller works inside the full saturation constraint given by (27). Figure 2 shows a scheme to represent the electric current used for the compensation and its interaction with the nominal control.

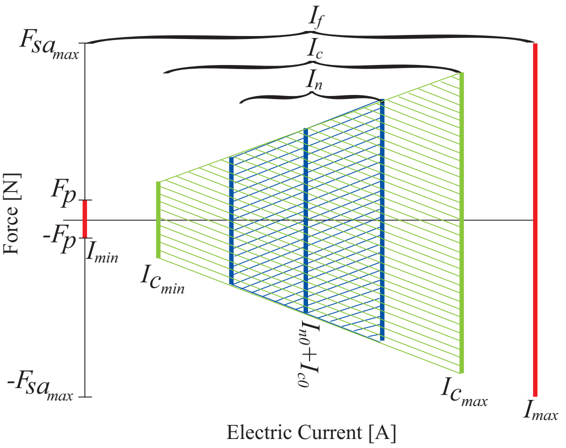

Fig. 2. Force - electric current diagram in the semi-active damper.

2) LPV model formulation: Following [6], an *LPV* controller of each *QoV* equipped with a semi-active damper is designed to handle the non-linearity of the damper model and the dissipativity constraint (represented by the maximum limit of the input electric current I_f). The considered model is given by [6]:

$$\begin{cases} \dot{x}_{lpv} = \mathcal{A}_{lpv}\left(\rho_1^*, \rho_2^*\right) x_{lpv} + \mathcal{B}_{1_{lpv}} u_c + \mathcal{B}_{2_{lpv}} z_r \\ y_{lpv} = \mathcal{C}_{lpv} x_{lpv} \end{cases} \tag{30}$$

where,

$$x_{lpv} = \begin{bmatrix} x \\ x_f \end{bmatrix}^T, \mathcal{A}_{lpv}\left(\rho_1^*, \rho_2^*\right) = \begin{bmatrix} A + \rho_2^* B_{I0} C_{I0} & \rho_1^* B_I C_f \\ 0_{1\times 4} & A_f \end{bmatrix},$$
$$\mathcal{B}_{1_{lpv}} = \begin{bmatrix} 0_{4\times 1} \\ B_f \end{bmatrix}, \mathcal{B}_{2_{lpv}} = \begin{bmatrix} B_r \\ 0 \end{bmatrix}, \mathcal{C}_{lpv} = \begin{bmatrix} C \\ 0 \end{bmatrix}^T$$

x is the state vector of a *QoV* with matrices A, B_I, B_{I0}, B_r, C and C_{I0} that include the *MR* damper dynamics [6], with z_{def} and $\dot{z}_{def}$ as output; x_f, A_f, B_f, C_f are the state and matrices of a representation of the low-pass filter $W_{filter} = \omega_f/(s+\omega_f)$, added to the plant to make the control input matrices parameter independent, [14].

Remark 1: The *LPV* model in (30) is based on the non-linearities introduced in eq. (1d), (2), (3) and (4) for a single corner, after some mathematical transformations as in [6].

3) LPV/$\mathcal{H}_\infty$ control synthesis: The extension to the method in [6] lies into the parameter ρ_2^* that incorporates the damping force used to compensate the faulty damper. The control design satisfies the actuator constraints. Indeed, depending on the value of the fault, the semi-active suspension is adapted to meet the required performances. The corresponding generalized plant $\Sigma(\rho_1^*, \rho_2^*)$ is given by:

$$\Sigma := \begin{cases} \dot{\xi}_{lpv} = \mathcal{A}_{lpv}(\rho_1^*, \rho_2^*)\xi_{lpv} + \mathcal{B}_{1_{lpv}} u_c + \tilde{\mathcal{B}}_{2_{lpv}} \tilde{w} \\ y_{lpv} = \mathcal{C}_{lpv}\xi_{lpv} \\ z = \mathcal{C}_z \xi_{lpv} + \mathcal{D}_{1_z} u_c + \mathcal{D}_{2_z} \tilde{w} \end{cases} \tag{31}$$

where z=$[z_1\ z_2]^T$, $\tilde{w}$=$W_{z_r}^{-1} z_r$, y=$[z_{def}\ \dot{z}_{def}]^T$, u_c=$u^{\mathcal{H}_\infty}$ and ξ_{lpv} includes the state variables of the *QoV* model and weighting functions, see details in [6].

C. Problem solution

The *LPV/$\mathcal{H}_\infty$* controller is synthesized in the *LMI* framework for polytopic systems; all varying parameters are bounded in $\rho_1^* \in [-1, 1]$ and $\rho_2^* \in [0, 3]$.

The resulting global *LPV/$\mathcal{H}_\infty$* controller is a convex combination of the local controllers obtained by solving the set of *LMIs* only on each vertex of the polytope formed by the limit values of the varying parameters. The stability will be guaranteed for all trajectories of the varying parameters, even if an extra compensation force is demanded.

Remark 2: Since 2 varying parameters are used, each corner considers a polytope of 4 vertices (4 local controllers, more details on *LPV* robust control design in [15]).

V. Simulation Results

Two scenarios have been considered to evaluate the proposed *FTC* in Matlab/Simulink™, by using the model (1).

A. Test #1: Without excitation of the roll angle.

The vehicle is driven at 30 km/h. The road profile is modeled as a *Chirp* signal (0.5 - 8 Hz) with magnitude 0.05 m on the left/right sides. Because the left and right wheels are synchronized, the roll angle is not excited. The fault at $t = 0s$ is a lack of force of 50% ($\alpha = 0.5$) on the front-left semi-active damper, Fig. 3a. By using the proposed *FDI* approach described in Section III, Fig. 3b shows that the fault estimation $\hat{F}_{\delta_1}$ is perfectly unsensitive to the road profile (an horizon $s = 1$ was used).

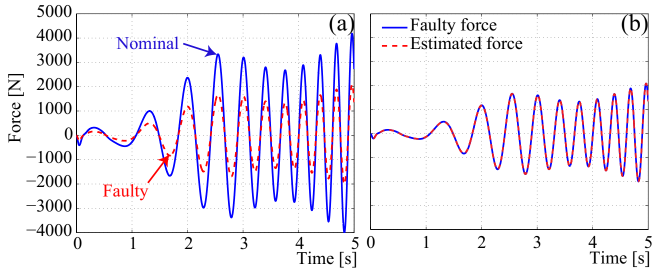

Fig. 3. Comparison between the nominal and faulty forces when $\alpha = 0.5$ (a), and fault estimation performance (b).

Figure 4 shows the performance of the proposed fault-tolerant semi-active suspension system depicted in Section IV versus an uncontrolled suspension (passive suspension) and a controlled suspension that does not include the fault compensation strategy (*LPV nominal control*); *Susp. Free of Faults* refers to the simulation of the *LPV/$\mathcal{H}_\infty$* suspension control when no fault is considered (in this case the roll angle is zero). By using the Root Mean Square (*RMS*) value of the roll angle of the vehicle as a comfort performance index, normalized w.r.t. the uncontrolled suspension, the proposed *FTC* improves the vehicle comfort by 82% and the nominal control by 22%, i.e. the reconfiguration strategy allows an improvement of 60% of comfort, see bar graph in Fig. 4.

Figure 5a shows how the semi-active damping force increases at the front-right actuator to compensate for the missing force of the faulty damper. The semi-activeness constraint of the *MR* damper is then ensured with the proposed *FTC* system. Figure 5b shows that the nominal *LPV* controller, whose performance is good when the suspension is free of faults, operates between 0 and 2 A without the compensation; when the compensation is used, the *LPV* controller utilizes

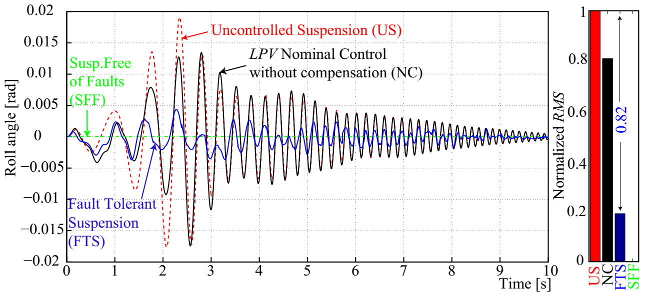

Fig. 4. Performance of the *FTC* in the test #1 (left) and comparison w.r.t. the uncontrolled suspension by using a normalized *RMS* (right).

the full range of actuation to generate more damping force and satisfies the saturation constraint of the damper (0-6 A).

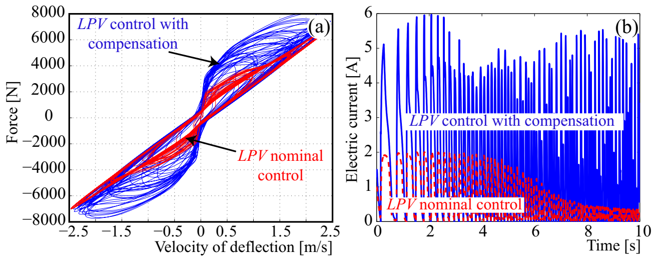

Fig. 5. Semi-active damping force in the front-right corner with and without fault compensation (a), and the corresponding controller output (b).

B. Test #2: Roll angle excitation

The vehicle is driven at 30 km/h in a straight way and runs over 5 sequential bumps of 5 cm, and the left and right bumps are dephased half wheelbase in order to excite the roll angle. This test allows to analyze the performances of the *LPV* nominal control and *LPV-FTC* facing to an uncontrolled suspension system. The fault at $t = 1.5s$ is a lack of force of 80% ($\alpha = 0.2$) on the front-left *MR* damper.

Figure 6 shows that before the fault occurs ($t = 1.5s$), the roll performance of the *LPV* controller (with or without fault compensation) is better than the passive suspension, i.e. the robust performance for comfort is 20% improved w.r.t. the passive suspension. By using the *RMS* index, the *FTC* reduces the roll angle by 43% w.r.t the uncontrolled suspension and by 24% w.r.t. the controller without fault-tolerance, Fig. 6.

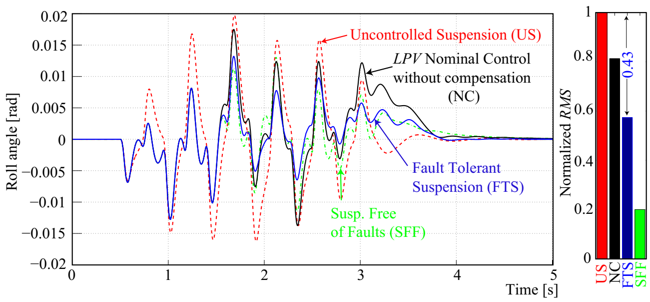

Fig. 6. Performance of the *FTC* in the test #2 (left) and comparison w.r.t. the uncontrolled suspension by using a normalized *RMS* (right).

VI. Conclusion

An effective Fault Tolerant Control (*FTC*) in a semi-active suspension system has been proposed for actuator faults. When a fault occurs, such as oil leakages in a damper, the *FTC* system is designed to compensate for the lack of damping force by using the other three healthy shock absorbers. At a high level, a *Fault Detection and Isolation* module based on parity-space is used to detect and isolate the faulty damper, and a fault reconfiguration strategy computes the compensation force to be added to the healthy dampers. Each corner has a fault estimator and an *LPV* controller for comfort and road holding; the *LPV* control design considers the compensation force as a varying parameter in order to take into account the semi-active damper constraints.

Simulation results show the robust performance of the nominal *LPV* controller w.r.t. a passive suspension, when the system is free of faults. When an oil leakage in the front-left damper is present, the proposed *FTC* system improves the comfort up to 60% and 82% w.r.t. the controlled (without compensation) and uncontrolled suspension system, respectively.

References

[1] C. Poussot-Vassal, C. Spelta, O. Sename, S.M. Savaresi and L. Dugard, "Survey and Performance Evaluation on some Automotive Semi-Active Suspension Control Methods: A Comparative Study on a Single-Corner Model", *Annual Reviews in Control*, 36(1), 2012, pp: 148-160.

[2] S. Savaresi, C. Poussot-Vassal, C. Spelta, O. Sename and L. Dugard, "Semi-Active Suspension Control for Vehicles", *Elsevier - Butterworth Heinemann*, 2010

[3] A.L. Do, C. Spelta, S.M. Savaresi, O. Sename, L. Dugard and D. Delvecchio, "An LPV Control Approach for Comfort and Suspension Travel Improvements of Semi-Active Suspension Systems", *49th IEEE Conf. on Decision and Control*, USA, 2010, pp: 5560–5565.

[4] Y. Zhang and J. Jiang, "Bibliographical Review on Reconfigurable Fault-tolerant Control Systems", *Annual Reviews in Control*, vol. 32, 2008, pp: 229-252.

[5] D. Fischer and R. Isermann, "Mechatronic Semi-Active and Active Vehicle Suspensions", *Control Eng Practice*, vol. 12, 2004, pp: 1353–1367.

[6] A.L. Do, O. Sename and L. Dugard, "LPV Modelling and Control of Semi-Active Dampers in Automotive Systems", *Control of Linear Parameter Varying Systems with Applications*, Mohammadpour and Scherer (Eds.), Springer, 2012, ch.15, pp: 381-411.

[7] P. Gáspár, Z. Szabó and J. Bokor, "LPV Design of Fault-tolerant Control for Road Vehicles", *IEEE Conf. on Control and Fault Tolerant Systems*, France, 2010, pp: 807–812.

[8] A. Chamseddine and H. Noura, "Control and Sensor Fault Tolerance of Vehicle Active Suspension", *IEEE Trans. on Control Systems Tech.*, 16(3), 2008, pp: 416–433.

[9] J.C. Tudón-Martínez, S. Varrier, R. Morales-Menendez, R. Ramirez-Mendoza, D. Koenig, J.J. Martinez, O. Sename, "Fault Tolerant Control with Additive Compensation for Faults in an Automotive Damper", *10th IEEE ICNSC*, France, 2013.

[10] D. Sammier, O. Sename and L. Dugard, "Skyhook and H_∞ Control of Semi-Active Vehicle Suspensions: Some Practical Aspects", *Vehicle System Dynamics*, vol. 39, 2003, pp: 279–308.

[11] S. Guo, S. Yang and C. Pan, "Dynamical Modeling of Magneto-rheological Damper Behaviors", *J. of Intell. Mater., Syst. and Struct.*, vol. 17, 2006, pp: 3–14.

[12] J.J. Lozoya-Santos, R. Morales-Menendez, R.A. Ramirez-Mendoza, J.C Tudón-Martínez, O. Sename and L. Dugard, "Magnetorheological Damper - An Experimental Study", *J. of Intelligent Mater. Syst. and Struct.*, vol. 23, 2012, pp: 1213–1232

[13] S. Varrier, D. Koenig and J.J. Martinez, "Robust Fault Detection for Vehicle Lateral Dynamics", *51st IEEE Conf. on Decision and Control*, USA, 2012, pp: 4366–4371.

[14] C. Poussot-Vassal, O. Sename, L. Dugard, P. Gáspár, Z. Szabó and J. Bokor, "A New Semi-Active Suspension Control Strategy through LPV Technique", *Control Eng. Practice*, vol. 16, 2008, pp: 1519–1534.

[15] C. Scherer, P. Gahinet and M. Chilali, "Multiobjective Output-Feedback Control Via LMI Optimization", *IEEE Trans. on Automatic Control*, 42(7), 1997, pp: 896–911.